HANDLING UNCERTAINTY BY DECONTEXTUALIZING ESTIMATED INTERVALS

Vagan Terziyan*, Seppo Puuronen, Helen Kaikova***

**Kharkov State Technical University of Radioelectronics,14 Lenina Av.,
310166 Kharkov, Ukraine; e-mail: vagan@jytko.jyu.fi, helen@kture.cit-ua.net
**University of Jyvaskyla, P.O.Box 35,
FIN-40351 Jyvaskyla, Finland, e-mail: sepi@jytko.jyu.fi*

Abstract

This paper considers the context sensitive approach to handle interval knowledge acquired from multiple knowledge sources. Each source gives its estimation of the value of some parameter *x*. The goal is to process all the intervals in a context of trends caused by some noise and derive resulting estimation that is more precise than the original ones and also takes into account the context noise. The main assumption used is that if a knowledge source guarantees smaller measurement error (estimated interval is shorter), then this source in the same time is more resistant against the effect of noise. This assumption allows us to derive and process trends among intervals and end up to shorter resulting estimated interval than any of the original ones.

Keywords: knowledge acquisition, multiple experts, noisy context, interval estimations, trends

1 Introduction

It is generally accepted that knowledge has a contextual component. Acquisition, representation, and exploitation of knowledge in context would have a major contribution in knowledge representation, knowledge acquisition, and explanation [3]. It is noticed in [4] that knowledge-based systems do not use correctly their knowledge. Knowledge being acquired from human experts does not usually include its context.

Contextual component of knowledge is closely connected with eliciting expertise from one or more experts in order to construct a single knowledge base (or, for example as in [2], for cooperative building of explanations). If more than one expert are available, one must either select the opinion of the best expert or pool the experts' judgments [14]. It is assumed here that when experts' judgments are pooled, collectively they offer sufficient cues leading to smaller uncertainty.

All information about the real word comes from two sources: from measurements, and from experts [9]. Measurements are not absolutely accurate. Every measurement instrument usually has the guaranteed upper bound of the measurement error. The measurement result is expected to lie in the interval around the actual value. This inaccuracy leads to the need to estimate the resulting inaccuracy of data processing. When experts are used to estimate the value of some parameter, intervals are commonly used to describe degrees of belief [14]. Experts are often uncertain about their degrees of belief making far larger estimation errors than the boundaries accepted by them as feasible [7]. In both cases we deal with interval uncertainty, i.e. we do not know exact values of parameters, only intervals where the values of these parameters belong to. A number of methods to define operations on intervals that produce guaranteed precision have been developed in [12], [13], [10], and [1] among others.

In many real life cases there is also some noise which does not allow direct measurement of parameters. To get rid of this noise it is necessary to subtract its value from the result of measurement. The noise can be considered as an undesirable effect to the evaluation of a parameter in the context. The subtraction of the noise in this sense has certain analogy with the decontextualization [11], [8], [5]. When effect of noise is not known it might be estimated using several coexisting knowledge sources. Some geometrical heuristics were used in [6] to solve this problem without enough mathematical justification. It is natural to assume that different measurement instruments as well as different experts possess different resistance against the influence of noise. Using measurements from several different instruments as well as estimations from multiple experts we try to discover the effect caused by noise and thus be able to derive the decontextualized measurement result.

This paper considers a context sensitive approach to handle interval knowledge acquired from multiple knowledge sources. Each source is assumed to give its evaluation, i.e. an estimated interval to which the value of a parameter *x* belongs. The goal is to process all the given intervals in the contexts of trends and derive more precise estimation of the value of parameter from them. The quality of each source is considered from two points of view: first, the value of guaranteed upper bound of measurement error, and second, the value of a resistance against a noise. These are assumed to occur together. The main assumption in this paper is that if a knowledge source guarantees lower upper bound of the measurement error, then the source in the same time is more resistant against the effect of noise. This assumption allows us to derive different trends that result to shorter intervals for the value of the parameter *x*. These are then combined to more precise estimation of this value.

In chapter 2 we present our decontextualization process and some of it main characteristics in the case of one trend. Next chapter discusses about one way to formulate groups of trends and its relation to decontextualization process. Chapter 4 discusses combining results of several trends into one resulting interval. The last chapter includes very short conclusion.

2 Decontextualization

In this chapter we consider a decontextualization process that is used to improve interval estimation by processing recursively more bounded intervals against less bounded ones.

Let there be *n* knowledge sources (human beings or measurement instruments) which are asked to make estimations of the value of a parameter *x*. Each knowledge source *i, i=1,…,n* gives his estimation as a closed interval $L_{[a_i,b_i]}$, $a_i < b_i$ into which he is sure that the value of the parameter belongs to.

Definition 2.1:

The *range* of a parameter *x* is the length $b_0 - a_0$ of the interval $L_{[a_0,b_0]}$, which includes all possible intervals $L_{[a_i,b_i]}$, $i = 1,\ldots,n$ of this parameter estimation.

Let us assume that all the knowledge sources are effected by the same misleading noise in the context of estimation. Of course different knowledge sources are effected by such a noise in a different way. The main assumption that we use in this paper is that: if a knowledge source guarantees smaller measurement error (interval estimation is more narrow), then this source is also more resistant against the effect of noise. This assumption also means that the estimated value given by more precise knowledge source is supposed to be closer to the actual value of the parameter *x*. This assumption is used when we derive trends of intervals towards the actual value of the parameter *x*.

The process advances decontextualizing an interval in the context of another interval (the step of decontextualization process) pairwise beginning from the shortest and second shortest intervals. The next step of decontextualization process is made decontextualizing the resulting interval of the first step in the context of the third shortest original interval. This is continued until all original intervals have been participated the process. The result of the last step is the result of the whole decontextualization process.

Definition 2.2:

The *uncertainty* u_i of an interval $L_{[a_i,b_i]}$ of parameter estimation is equal to the length of the interval:

$$u_i = b_i - a_i, \ i=1,\ldots,n.$$

To be precise it is necessary to mention that in a general case the value of uncertainty should be standardized with the range of the parameter estimated, like the following:

$$u_i^{st} = \frac{b_i - a_i}{b_0 - a_0}, \; i=1,\ldots,n.$$

In this paper, however, we use and compare different estimations of the same parameter within the same range. That is why it is not essential to standardize a value of uncertainty and we can use the Definition 2.2 working with uncertainty.

Definition 2.3:

The *quality* q_i of an interval $L_{[a_i, b_i]}$ is the reverse of its uncertainty, i.e.: $q_i = \dfrac{1}{u_i}$, $i=1,\ldots,n$.

2.1 Operating with two intervals

Definition 2.4:

The step of the decontextualization between intervals $L_{[a_i, b_i]}$ and $L_{[a_j, b_j]}$, $u_i \neq u_j$, $i=1,\ldots,n$ is:

$$L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{\left[a_i + \frac{u_i^2 \cdot (a_i - a_j)}{u_j^2 - u_i^2},\; b_i + \frac{u_i^2 \cdot (b_i - b_j)}{u_j^2 - u_i^2}\right]}.$$

The above formulas for calculating the resulting interval was selected because they satisfy three main requirements:

- the resulting interval should be shorter than the original ones,
- the longer the original intervals are the longer should the resulting interval be, and
- shorter of the two intervals should locate closer the resulting interval than the longer one.

In the following we will prove that the selected formulas fulfill these three main requirements.

The following theorem defines the relationships between the uncertainties of the original and the resulting intervals.

Theorem 2.1:

Let it be that $L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{[a_{res},\; b_{res}]}$, where a_{res} and b_{res} are as in the right hand part of the Definition 2.4.

Then: (a) $u_{res} = \dfrac{u_i \cdot u_j}{u_i + u_j}$, (b) $u_{res} < u_i$, (c) $u_{res} < u_j$, and (d) $q_{res} = q_i + q_j$.

Proof:

a) According to the Definition 2.4: $a_{res} = a_i + \dfrac{u_i^2 \cdot (a_i - a_j)}{u_j^2 - u_i^2}$ and $b_{res} = b_i + \dfrac{u_i^2 \cdot (b_i - b_j)}{u_j^2 - u_i^2}$.

Definition 2.2. gives us that:

$$u_{res} = b_{res} - a_{res} = (b_i - a_i) + \frac{u_i^2}{u_j^2 - u_i^2} \cdot ((b_i - a_i) - (b_j - a_j)) = u_i + \frac{u_i^2}{u_j^2 - u_i^2} \cdot (u_i - u_j) =$$

$$= u_i - \frac{u_i^2}{u_i + u_j} = \frac{u_i \cdot u_j}{u_i + u_j};$$

Thus: $\underline{u_{res} = \dfrac{u_i \cdot u_j}{u_i + u_j}}$.

b) Let us suppose that: $u_{res} \geq u_i$; then according to (a) we receive: $\dfrac{u_i \cdot u_j}{u_i + u_j} \geq u_i \Rightarrow \dfrac{u_i + u_j}{u_j} \leq 1 \Rightarrow u_i \leq 0$, which contradicts the Definition 2.2. Thus: $\underline{u_{res} < u_i}$.

c) Prove is similar as for (b).

d) From the Definition 2.3. it results that: $u_i = \dfrac{1}{q_i}$, $i=1,\ldots,n$.

Applying (a) we receive that:

$$\frac{1}{q_{res}} = \frac{\dfrac{1}{q_i} \cdot \dfrac{1}{q_j}}{\dfrac{1}{q_i} + \dfrac{1}{q_j}} = \frac{1}{q_i + q_j};$$

Thus: $\underline{q_{res} = q_i + q_j}$.

Theorem 2.2:

Let it be that: $L_{[a_i,b_i]}^{L_{[a_j,b_j]}} = L_{[a_{res_1},\ b_{res_1}]}$, and $L_{[a_i,b_i]}^{L_{[a_k,b_k]}} = L_{[a_{res_2},\ b_{res_2}]}$, where a_{res_1}, a_{res_2}, b_{res_1}, and b_{res_2} are as in the right hand part of Definition 2.4.

Let it be that: $u_j < u_k$. Then: $u_{res_1} < u_{res_2}$.

Proof:

$$u_j < u_k \Rightarrow u_i u_j < u_i u_k \Rightarrow u_i u_j + u_j u_k < u_i u_k + u_j u_k \Rightarrow$$

$$\Rightarrow (u_i + u_k) u_j < (u_i + u_j) u_k \Rightarrow \frac{u_j}{u_i + u_j} < \frac{u_k}{u_i + u_k} \Rightarrow \frac{u_i \cdot u_j}{u_i + u_j} < \frac{u_i \cdot u_k}{u_i + u_k} \Rightarrow u_{res_1} < u_{res_2}.$$

Theorem 2.3:

Let it be that: $L_{[a_j,b_j]}^{L_{[a_i,b_i]}} = L_{[a_{res_1},b_{res_1}]}$, and $L_{[a_k,b_k]}^{L_{[a_i,b_i]}} = L_{[a_{res_2},\ b_{res_2}]}$, where a_{res_1}, a_{res_2}, b_{res_1}, and b_{res_2} are as in the right hand part of the Definition 2.3. Let it be that $u_j < u_k$. Then: $u_{res_1} < u_{res_2}$.

Proof: Similarly as Theorem 2.2.

One possible interpretation of the step of decontextualization formula is based on an extrapolation of the decontextualized value using interval functions. Extrapolation is based on assumption of linearity of these functions within one step of decontextualization.

The two linear functions are considered: $a = f(u)$, that connects points $(u_i, a_i), (u_j, a_j)$, and $b = \varphi(u)$ that connects points $(u_i, b_i), (u_j, b_j)$ as it is shown in Figure 1.

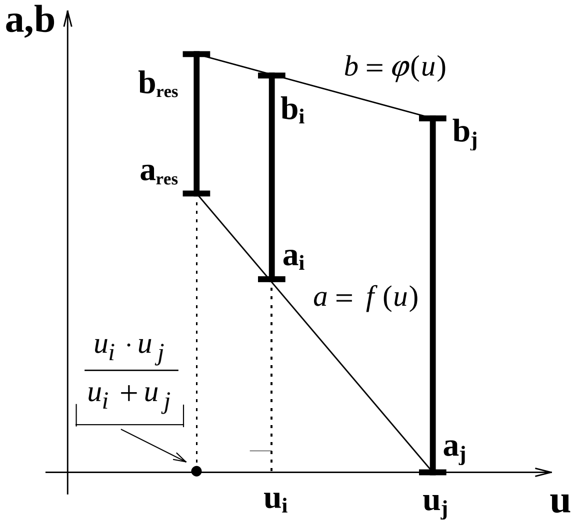

Figure 1: Deriving decontextualized interval by linear extrapolation

If we want to obtain values $f\left(\frac{u_i \cdot u_j}{u_i + u_j}\right)$ and $\varphi\left(\frac{u_i \cdot u_j}{u_i + u_j}\right)$, then we should solve equations:

$$\frac{u_i - \frac{u_i \cdot u_j}{u_i + u_j}}{u_j - u_i} = \frac{a_i - a_{res}}{a_j - a_i}; \quad \frac{u_i - \frac{u_i \cdot u_j}{u_i + u_j}}{u_j - u_i} = \frac{b_i - b_{res}}{b_j - b_i}.$$

The solution of these equations gives us the following values:

$$a_{res} = a_i + \frac{u_i^2 \cdot (a_i - a_j)}{u_j^2 - u_i^2}; \; b_{res} = b_i + \frac{u_i^2 \cdot (b_i - b_j)}{u_j^2 - u_i^2},$$

which are exactly the same as if we make one step of decontextualization process accordingly to the Definition 2.4. Also this fact motivates the selection of the formula in the Definition 2.3 from the point of view of the possibility to obtain result of decontextualization using linear extrapolation.

Physical interpretation of the decontextualization formula is shown in Figure 2. It represents the uncertain opinion as certain resistance to correct parameter estimation. However the more opinions we consider (parallel resistances in Figure 2) the more exact estimation we will obtain due to the formula of parallel resistances.

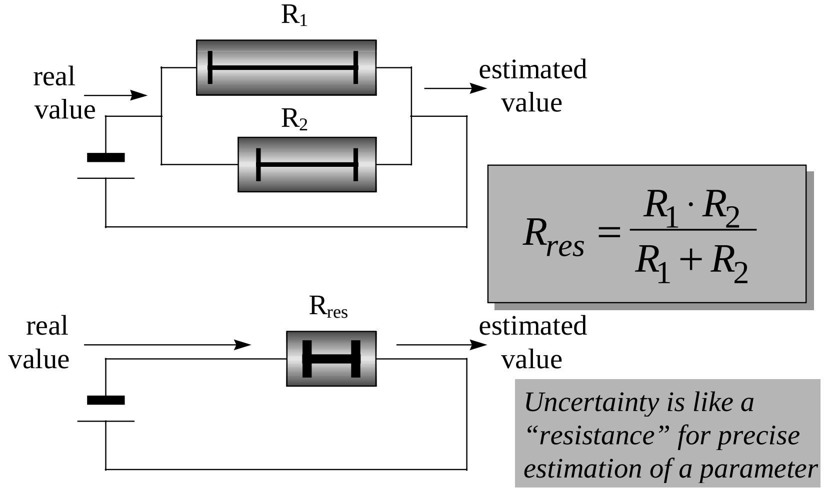

Figure 2: Physical interpretation of decontextualization

Theorem 2.4:

$$L_{[a_i,b_i]}^{L_{[a_j,b_j]}} = L_{[a_j,b_j]}^{L_{[a_i,b_i]}}.$$

Proof:

$$L_{[a_i,b_i]}^{L_{[a_j,b_j]}} = L_{\left[a_i+\frac{u_i^2\cdot(a_i-a_j)}{u_j^2-u_i^2},\ b_i+\frac{u_i^2\cdot(b_i-b_j)}{u_j^2-u_i^2}\right]} = L_{\left[\frac{u_j^2\cdot a_i-u_i^2\cdot a_j}{u_j^2-u_i^2},\ \frac{u_j^2\cdot b_i-u_i^2\cdot b_j}{u_j^2-u_i^2}\right]} = L_{\left[\frac{u_i^2\cdot a_j-u_j^2\cdot a_i}{u_i^2-u_j^2},\frac{u_i^2\cdot b_j-u_j^2\cdot b_i}{u_i^2-u_j^2}\right]} =$$

$$= L_{\left[a_j+\frac{u_j^2\cdot(a_j-a_i)}{u_i^2-u_j^2},\ b_j+\frac{u_j^2\cdot(b_j-b_i)}{u_i^2-u_j^2}\right]} = L_{[a_j,b_j]}^{L_{[a_i,bi]}}.$$

Definition 2.5:

Let us have two interval opinions $L_{[a_i,b_i]}$ and $L_{[a_j,b_j]}$, $i, j = 1,\ldots,n$.

The *distance* between these opinions is as follows:

$$D(L_{[a_i,b_i]}, L_{[a_j,b_j]}) = \max(abs(a_j - a_i), abs(b_j - b_i)).$$

Theorem 2.5:

If it holds that: $L_{[a_i,b_i]}^{L_{[a_j,b_j]}} = L_{[a_{res},\ b_{res}]}$, and $u_i < u_j$, then: $D(L_{[a_{res},b_{res}]}, L_{[a_i,b_i]}) < D(L_{[a_{res},b_{res}]}, L_{[a_j,b_j]})$.

Proof:

Using Definitions 2.4 and 2.5 we receive:

$$(D(L_{[a_{res},b_{res}]}, L_{[a_i,b_i]}) = \max(abs(a_i - a_{res}), abs(b_i - b_{res})) =$$

$$= \max(abs(\frac{u_i^2\cdot(a_j-a_i)}{u_j^2-u_i^2}), abs(\frac{u_i^2\cdot(b_j-b_i)}{u_j^2-u_i^2}))) < (\max(abs(\frac{u_j^2\cdot(a_j-a_i)}{u_j^2-u_i^2}), abs(\frac{u_j^2\cdot(b_j-b_i)}{u_j^2-u_i^2})) =$$

$$= \max(abs((a_j-a_i)-\frac{u_i^2\cdot(a_i-a_j)}{u_j^2-u_i^2}), abs((b_j-b_i)-\frac{u_j^2\cdot(b_i-b_j)}{u_j^2-u_i^2})) =$$

$$= \max(abs(a_j - a_{res}), abs(b_j - b_{res})) = D(L_{[a_{res},b_{res}]}, L_{[a_j,b_j]})).$$

2.2 Operating with several intervals

The process of decontextualization with several intervals was described in the beginning of this chapter. We describe now this step by step process formally.

Let there be n intervals $L_{[a_i,b_i]}$, $i = 1,\ldots,n$, $n \geq 2$, $u_i < u_{i+1}$, $i = 1,\ldots,n-1$. The resulting interval:

$$L_{[a_{res},b_{res}]} = L_{[a_1,b_1]}^{L_{[a_2,b_2]}^{L_{\ldots}^{L_{[a_n,b_n]}}}}$$

can be calculated recursively as follows:

$$L_{[a_{res_1},b_{res_1}]} = L_{[a_1,b_1]}; \quad L_{[a_{res_i},b_{res_i}]} = L_{[a_{res_{i-1}},b_{res_{i-1}}]}^{L_{[a_i,b_i]}}, i = 2,\ldots,n; \quad L_{[a_{res},b_{res}]} = L_{[a_{res_n},b_{res_n}]}.$$

Theorem 2.6:

If it holds that:

$$(L_{[a_i,b_i]}^{L_{[a_j,b_j]}})^{L_{[a_k,b_k]}} = L_{[a_{res'},\ b_{res'}]}, \quad \text{and } L_{[a_i,b_i]}^{(L_{[a_j,b_j]}^{L_{[a_k,b_k]}})} = L_{[a_{res''},\ b_{res''}]}, \quad \text{then: } u_{res'} = u_{res''}.$$

Proof:

$$u_{res'} = \frac{\left(\frac{u_i \cdot u_j}{u_i + u_j}\right) \cdot u_k}{\left(\frac{u_i \cdot u_j}{u_i + u_j}\right) + u_k} =$$

$$= \frac{\frac{u_i \cdot u_j \cdot u_k}{u_i + u_j}}{\frac{u_i \cdot u_j + u_i \cdot u_k + u_j \cdot u_k}{u_i + u_j}} = \frac{u_i \cdot u_j \cdot u_k}{u_i \cdot (u_j + u_k) + u_j \cdot u_k} =$$

$$= \frac{\frac{u_i \cdot u_j \cdot u_k}{u_j + u_k}}{\frac{u_i \cdot (u_j + u_k) + u_j \cdot u_k}{u_j + u_k}} = \frac{u_i \cdot \left(\frac{u_j \cdot u_k}{u_j + u_k}\right)}{u_i + \left(\frac{u_j \cdot u_k}{u_j + u_k}\right)} = u_{res''}.$$

An example: Let us suppose that three knowledge sources 1, 2, and 3 evaluate the value of the attribute *x* to be in the following intervals:

$$L_{[a_1,b_1]} = L_{[9,12]}, L_{[a_2,b_2]} = L_{[6,11]}, L_{[a_3,b_3]} = L_{[0,10]}.$$

The above intervals are already in ascending order according to their uncertainties. The resulting interval is derived by the recursive procedure above:

$$L_{[a_{res_1},b_{res_1}]} = L_{[a_1,b_1]} = L_{[9,12]};$$

$$L_{[a_{res_2},b_{res_2}]} = L_{[a_{res_1},b_{res_1}]}^{L_{[a_2,b_2]}} = L_{[9,12]}^{L_{[6,11]}} = L_{[10.6875,12.5625]};$$

$$L_{[a_{res_3},b_{res_3}]} = L_{[a_{res_2},b_{res_2}]}^{L_{[a_3,b_3]}} = L_{[10.6875,12.5625]}^{L_{[0,10]}} \approx L_{[11.0769,12.6559]},$$

and thus: $L_{[a_{res},b_{res}]} = L_{[a_{res_3},b_{res_3}]} = L_{[11.0769,12.6559]}$.

The resulting interval with the original ones is shown in Figure 3.

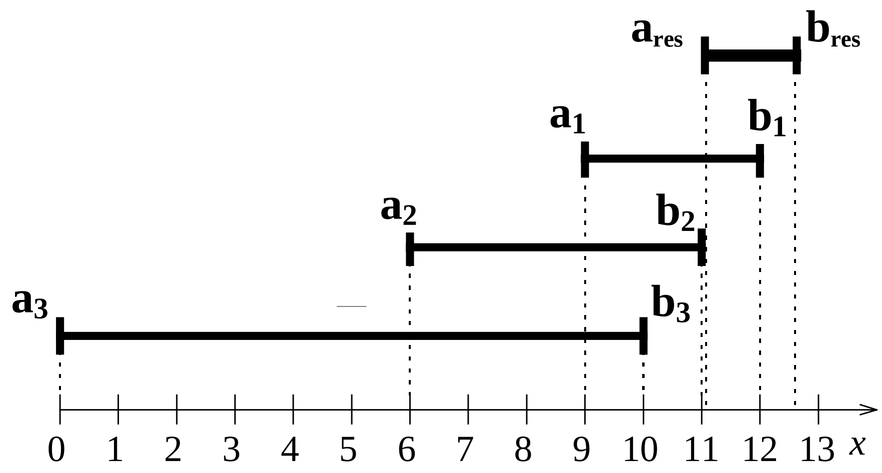

Figure 3: The resulting interval and the original ones in the example

One can see that the resulting interval has no common points with two of the three original intervals. This happens because the decontextualization process takes into account the trend caused by noise in the estimation context.

3 A Trend Classification

In this chapter we consider one classification of trends into seven different groups of trends. This classification is used to group together intervals based on the relations of their endpoints and we prove that the previous step of decontextualization (Definition 2.4) gives as a result an interval that belongs to the same group as the intervals participating into the decontextualization process.

Definition 3.1:

There are seven groups of trends named as trends with *direction* dir_k and *power* pow_k (marked $L_k^{dir_k\ pow_k}$) as presented in Table 1. Each pair of intervals $L_{[a_i,b_i]}, L_{[a_j,b_j]} \in L_{[a_0,b_0]}, i \neq j$, belonging to the same group $L_k^{dir_k\ pow_k}$ keep the sign of $\Delta a + \Delta b$, Δa, and Δb where $\Delta a = a_j - a_i$, $\Delta b = b_j - b_i$.

The direction of a trend group is: *left ('l')*, *center ('c')*, or *right ('r')*, and it is defined by the sign of $\Delta a + \Delta b$:

$$(\Delta a + \Delta b > 0) \Rightarrow dir_k = \text{'}l\text{'};\ (\Delta a + \Delta b = 0) \Rightarrow dir_k = \text{'}c\text{'};$$

$$(\Delta a + \Delta b < 0) \Rightarrow dir_k = \text{'}r\text{'}.$$

The power of a trend group is: *slow ('<')*, *medium ('=')*, or *fast ('>')* and it is defined by the signs of Δa and Δb by the following way:

$$((\Delta a < 0)\ and\ (\Delta b > 0)) \Rightarrow pow_k = \text{'<'};$$

$$((\Delta a = 0)\ or\ (\Delta b = 0)) \Rightarrow pow_k = \text{'='};$$

$$((\Delta a > 0)\ or\ (\Delta b < 0)) \Rightarrow pow_k = \text{'>'}.$$

In Figure 4 there are three examples of trend groups with left direction and power: slow (a), medium (b), and fast (c).

Table 1: Trends of uncertainty

Trend	Direction $\rightarrow$	left	central	right
Power $\downarrow$	Restrictions	$\Delta a + \Delta b > 0$	$\Delta a + \Delta b = 0$	$\Delta a + \Delta b < 0$
slow	$(\Delta a < 0) and (\Delta b > 0)$	$L^{l<}$	$L^{c<}$	$L^{r<}$
medium	$(\Delta a = 0) or (\Delta b = 0)$	$L^{l=}$	does not exist	$L^{r=}$
fast	$(\Delta a > 0) or (\Delta b < 0)$	$L^{l>}$	does not exist	$L^{r>}$

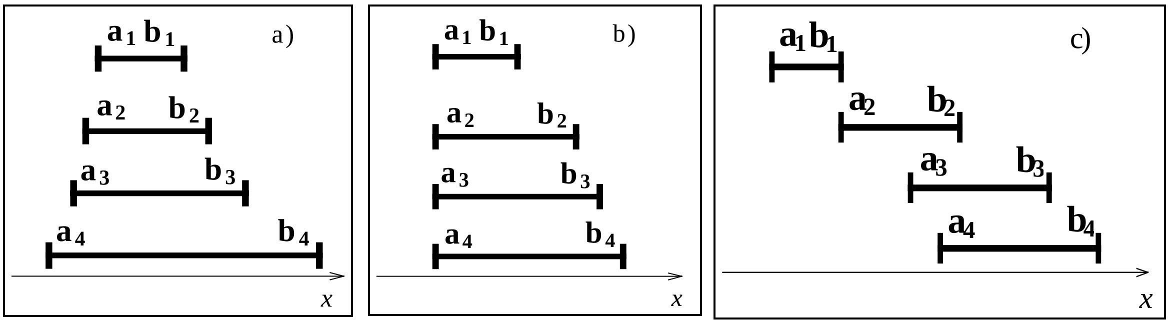

Figure 4: Left trend groups $L^{l<}, L^{l=}, L^{l>}$

Theorem 3.1:

When $L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{[a_{res},\ b_{res}]}$, then the interval $L_{[a_{res}, b_{res}]}$ belongs to the same trend group as the intervals $L_{[a_i, b_i]}, L_{[a_j, b_j]}$.

Proof:

The intervals $L_{[a_i, b_i]}, L_{[a_j, b_j]}$ belongs to the group according to the signs:

$$sign(\Delta a + \Delta b) = sign(a_j - a_i + b_j - b_i), \quad sign(\Delta a) = sign(a_j - a_i), \quad \text{and}$$
$$sign(\Delta b) = sign(b_j - b_i).$$

To prove the theorem it is necessary to prove that:

$$sign(\Delta a' + \Delta b') = sign(a_i - a_{res} + b_i - b_{res}) = sign(\Delta a + \Delta b);$$

$$sign(\Delta a') = sign(a_i - a_{res}) = sign(\Delta a)\,;\ sign(\Delta b') = sign(b_i - b_{res}) = sign(\Delta b)\,.$$

According the Definition 2.4:

$$a_{res} = a_i + \frac{u_i^2 \cdot (a_i - a_j)}{u_j^2 - u_i^2} = a_i - \frac{u_i^2}{u_j^2 - u_i^2} \cdot \Delta a\,;\ b_{res} = b_i - \frac{u_i^2}{u_j^2 - u_i^2} \cdot \Delta b\,.$$

Thus:

$$\left.\begin{array}{l} (0 < u_i < u_j) \Rightarrow (\dfrac{u_i^2}{u_j^2 - u_i^2} > 0); \quad \Delta a' = a_i - a_{res} = \dfrac{u_i^2}{u_j^2 - u_i^2} \cdot \Delta a \\ \Delta b' = b_i - b_{res} = \dfrac{u_i^2}{u_j^2 - u_i^2} \cdot \Delta b; \quad \Delta a' + \Delta b' = \dfrac{u_i^2}{u_j^2 - u_i^2} \cdot (\Delta a + \Delta b) \end{array}\right\} \Rightarrow$$

$$\underline{\Rightarrow sign(\Delta a') = sign(\Delta a)}; \quad \underline{sign(\Delta b') = sign(\Delta b)}; \quad \underline{sign(\Delta a' + \Delta b') = sign(\Delta a + \Delta b)}.$$

The Theorem 3.1 shows that the step of decontextualization gives resulting interval that belongs to the same group of trends as the original intervals participating the process.

4 Deriving a Resulting Interval in the Case of Several Trends

In a common case it is possible that several different trends can be derived from the same set of intervals. In this chapter we discuss one way of deriving resulting interval when there exist several trends among the original intervals.

Each pair of intervals can define a trend. We require that each pair participates once and only once in some of trends. This means that the number *m* of different trends cannot be more than the number of different interval pairs in the set of intervals $L = L_{[a_i, b_i]}$, $i = 1, \ldots n$ given as opinions by knowledge sources:

$$m \le C_n^2 .$$

Each trend can in general case include more than one interval and each interval can support several more than one trend.

Definition 4.1:

Let us suppose that the set L of interval opinions $L_{[a_i, b_i]}$, $i = 1, \ldots n$ is divided into m trends L_k, $k = 1, \ldots m$.

The *support* S_k for the trend L_k is calculated as follows:

$$S_k = q_{res}^k \cdot \sum_{\forall i, L_{[a_i, b_i]} \in L_k} \frac{1}{N_i},$$

where q_{res}^k is the quality of the result $L_{\left[a_{res}^k, b_{res}^k\right]}$,

N_i is the number of different trends that includes the opinion $L_{[a_i, b_i]}$.

As one can see the Definition 4.1 gives more support for the trend that includes more intervals and the support of each interval is divided equally between all the trends that include this interval.

Definition 4.2:

Let the set of original interval opinions $L = L_{[a_i, b_i]}$, $i = 1, \ldots, n$ consists of m different trends L_k, $k = 1, \ldots, m$ with their resulting interval opinions $L_{\left[a_{res}^k, b_{res}^k\right]}$ and support S_k.

Then the resulting opinion $L_{\left[a_{res}^L, b_{res}^L\right]}$ for the original set of interval opinions is derived using following formulas:

$$a_{res}^L = \frac{a_{res}^1 \cdot S_1 + a_{res}^2 \cdot S_2 + \ldots + a_{res}^m \cdot S_m}{S_1 + S_2 + \ldots + S_m};$$

$$b_{res}^L = \frac{b_{res}^1 \cdot S_1 + b_{res}^2 \cdot S_2 + \ldots + b_{res}^m \cdot S_m}{S_1 + S_2 + \ldots + S_m}.$$

Thus the resulting interval is expected to be closer to the result of those trends that have more support among the original set of intervals.

5 Conclusion

This paper discusses one approach to handle interval uncertainty in estimation of some domain parameter. The case is considered when the estimation is made by multiple knowledge sources in a context of a trend caused by possible noise. The approach is based on an assumption that if a knowledge source guarantees less measurement error (estimation interval is shorter), then this source in the same time is more resistant against the effect of possible noise. In this paper we discussed one way to decontextualize knowledge given under misleading noise when this basic assumption holds. We defined different groups of trends among estimated intervals. We introduced one way how to take into account several trends that exist among the original intervals when one resulting interval is produced. Further research is needed in different application areas to evaluate practical results of presented decontextualization process.

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