Terziyan, V., Puuronen, S., & Kaikova, H. (2000). An Interval Approach to Discover Knowledge from Multiple Fuzzy Estimations. In: B. Radig, H. Niemann, Y. Zhuravlev & I. Gourevitch (Eds.), *Pattern Recognition and Image Understanding. 5th Open German-Russian Workshop* (pp. 235-243). Amsterdam: IOS Press.

Let there be $n$ knowledge sources (human beings or measurement instruments) which are asked to make estimations of the value of a parameter $x$.

Each knowledge source $i$, $i=1,\ldots,n$ gives his estimation as a closed interval $L[a_i, b_i]$, $a_i < b_i$ into which he is sure that the estimated value belongs to.

***Definition:*** The ***uncertainty*** $u_i$ of an opinion $L[a_i, b_i]$ is equal to the length of the appropriate interval, i.e.: $u_i = b_i - a_i$, $i=1,\ldots,n$.

***Definition:*** The ***quality*** $q_i$ of an opinion $L[a_i, b_i]$ is the reverse of its uncertainty, i.e.: $q_i = \frac{1}{u_i}$, $i=1,\ldots,n$.

# Decontextualization with two Intervals

## (Main Requirements)

- the resulting interval should be shorter than the original ones;
- the longer the original intervals are the longer should the resulting interval be;
- shorter of the two intervals should locate closer the resulting interval than the longer one.

# Decontextualization with two Intervals

## (Basic Formula)

***Definition:*** The ***step of the decontextualization process*** between opinions $L_{[a_i, b_i]}$ and $L_{[a_j, b_j]}$, $u_i \neq u_j$, $i=1,\ldots,n$ produces the following interval:

$$L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{\left[a_i + \frac{u_i^2 \cdot (a_i - a_j)}{u_j^2 - u_i^2},\; b_i + \frac{u_i^2 \cdot (b_i - b_j)}{u_j^2 - u_i^2}\right]}$$

# Decreasing Uncertainty Theorem

***Theorem:*** Let it be that $L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{[a_{res},\ b_{res}]}$.

Then:

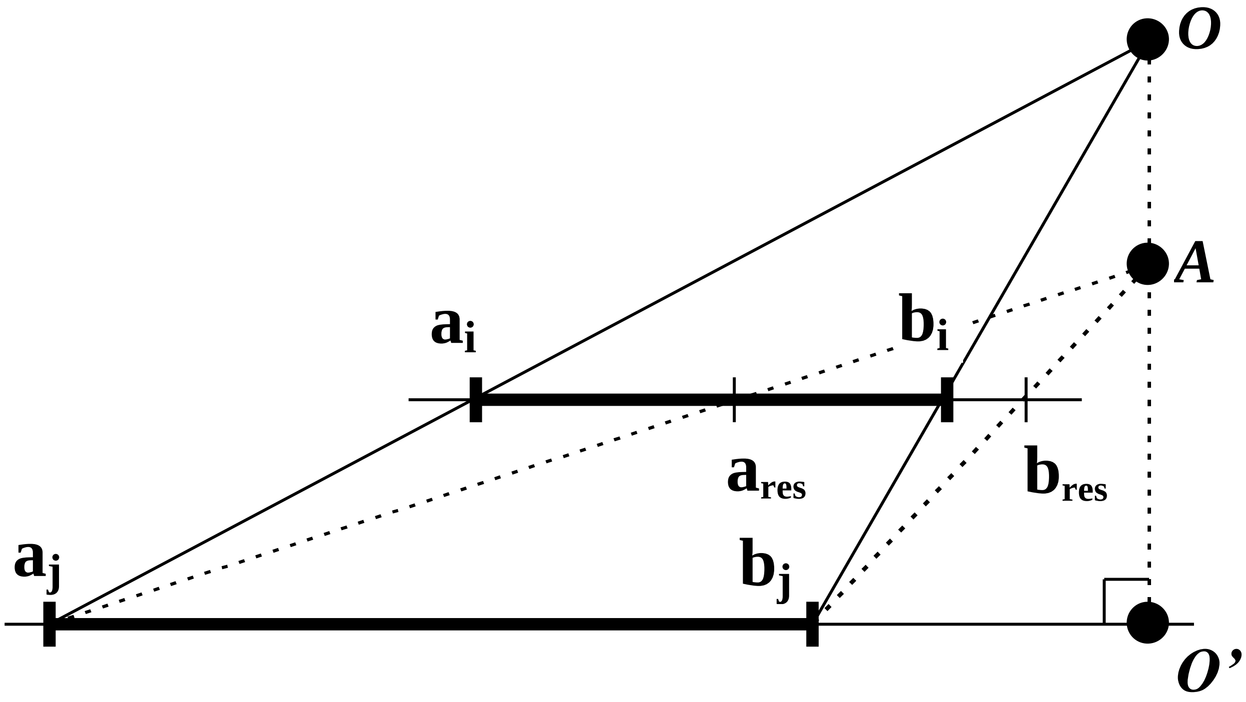
a) $$u_{res} = \frac{u_i \cdot u_j}{u_i + u_j},$$

b) $$u_{res} < u_i,$$

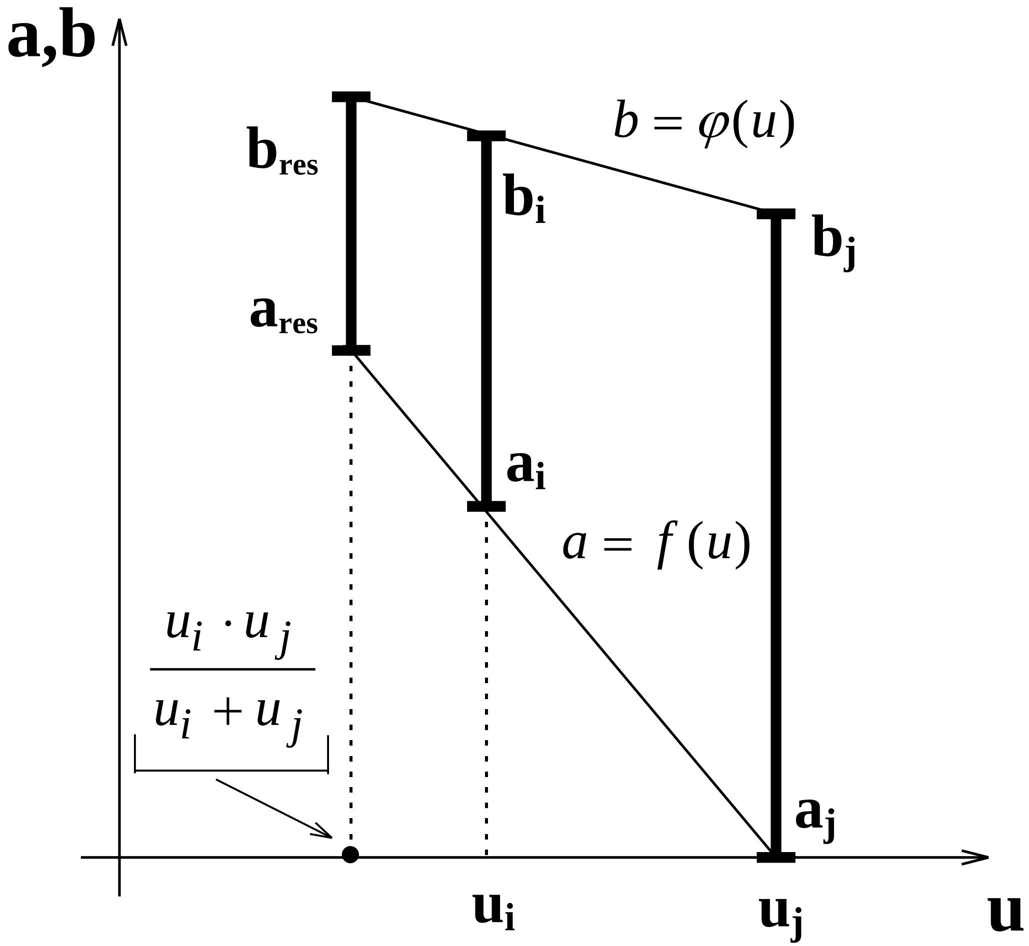
c) $$u_{res} < u_j,$$

d) $$q_{res} = q_i + q_j.$$

# Geometrical Interpretation of Decontextualization



# Extrapolation Interpretation of Decontextualization



# Distance Between Interval Opinions

***Definition:***

Let there are two interval opinions $L_{[a_i, b_i]}$ and $L_{[a_j, b_j]}$, $i, j = 1, \ldots, n$.

The ***distance*** between these opinions is as follows:

$$D(L_{[a_i, b_i]}, L_{[a_j, b_j]}) = \max(abs(a_j - a_i), abs(b_j - b_i)).$$

# Decontextualize Distance Theorem

If it holds that: $L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{[a_{res},\ b_{res}]}$ , and $u_i < u_j$ , then:

$$D(L_{[a_{res}, b_{res}]}, L_{[a_i, b_i]}) < D(L_{[a_{res}, b_{res}]}, L_{[a_j, b_j]}).$$

It means: shorter of the two intervals is located closer to the resulting interval than the longer one.

# Operating with Several Intervals

The ***resulting interval***: $L_{[a_{res},b_{res}]} = L_{[a_1,b_1]}^{L_{[a_2,b_2]}^{L_{\cdots}^{L_{[a_n,b_n]}}}}$ obtained as decontextualization of $n$ intervals $L_{[a_i,b_i]},\ i=1,\ldots,n$, $u_k < u_{k+1}$ can be calculated recursively as follows:

$$L_{[a_{res_1},b_{res_1}]} = L_{[a_1,b_1]};$$

$$L_{[a_{res_i},b_{res_i}]} = L_{[a_{res_{i-1}},b_{res_{i-1}}]}^{L_{[a_i,b_i]}}, i = 2,\ldots,n;$$

$$L_{[a_{res},b_{res}]} = L_{[a_{res_n},b_{res_n}]}.$$

# Uncertainty Associativity Theorem

If it holds that:

$$\left(L_{[a_i, b_i]}^{L_{[a_j, b_j]}}\right)^{L_{[a_k, b_k]}} = L_{[a_{res'},\ b_{res'}]},$$

and

$$L_{[a_i, b_i]}^{\left(L_{[a_j, b_j]}^{L_{[a_k, b_k]}}\right)} = L_{[a_{res''},\ b_{res''}]},$$

then: $$u_{res'} = u_{res''} .$$

# An Example:

Let us suppose that three knowledge sources ***1, 2,*** and ***3*** evaluate the attribute ***x*** to be within the following intervals:

$$L_{[a_1,b_1]} = L_{[9,12]},\ L_{[a_2,b_2]} = L_{[6,11]},\ L_{[a_3,b_3]} = L_{[0,10]}.$$

The resulting interval is derived by the recursive procedure:

$$L_{\left[a_{res_1},b_{res_1}\right]} = L_{[a_1,b_1]} = L_{[9,12]};\quad L_{\left[a_{res_2},b_{res_2}\right]} = L_{\left[a_{res_1},b_{res_1}\right]}^{L_{[a_2,b_2]}} = L_{[9,12]}^{L_{[6,11]}} = L_{[10.6875,12.5625]};$$

$$L_{\left[a_{res_3},b_{res_3}\right]} = L_{\left[a_{res_2},b_{res_2}\right]}^{L_{[a_3,b_3]}} = L_{[10.6875,12.5625]}^{L_{[0,10]}} \approx$$

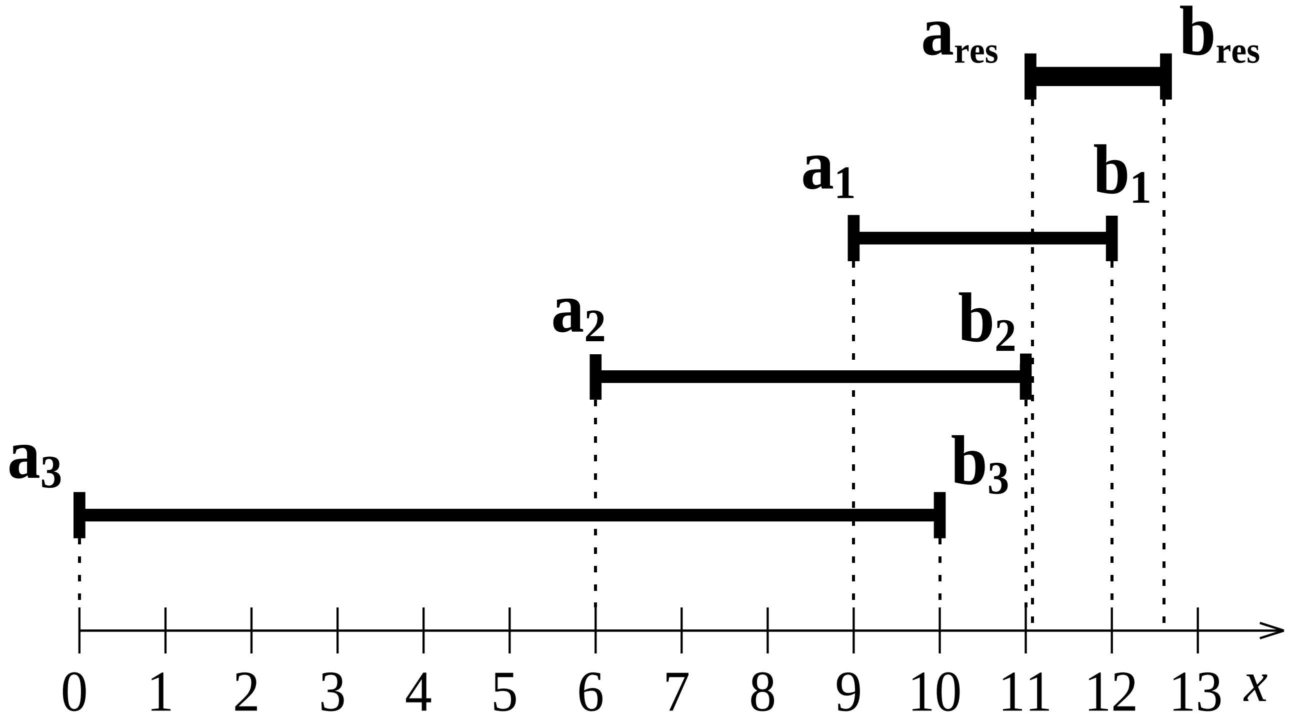
$$\approx L_{[11.0769,12.6559]},$$

and thus: $$L_{[a_{res},b_{res}]} = L_{\left[a_{res_3},b_{res_3}\right]} = L_{[11.0769,12.6559]}.$$

# An Example:

$$L_{[a_1,b_1]} = L_{[9,12]},\ L_{[a_2,b_2]} = L_{[6,11]},\ L_{[a_3,b_3]} = L_{[0,10]},$$

$$L_{[a_{res},b_{res}]} = L_{\left[a_{res_3},b_{res_3}\right]} = L_{[11.0769,12.6559]}$$



# A Trend of Uncertainty Group Definition

There are seven groups of ***trends*** $L_k^{dir_k\ pow_k}$ with ***direction*** $dir_k$ and ***power*** $pow_k$

Each pair of intervals $L_{[a_i,b_i]}, L_{[a_j,b_j]} \in L_{[a_0,b_0]}, i \neq j$, belonging to the same group $L_k^{dir_k\ pow_k}$ keep the sign of $\Delta a + \Delta b$, $\Delta a$, and $\Delta b$ where $\Delta a = a_j - a_i$, $\Delta b = b_j - b_i$.

Direction and power of a trend are defined by a concrete combination of signs for $\Delta a + \Delta b$, $\Delta a$, and $\Delta b$.

# Direction of a Trend Group

The ***direction of a trend group*** is:

***left (‘l’), centre (‘c’), right (‘r’),***

and it is defined by the sign of $\Delta a + \Delta b$ :

$$(\Delta a + \Delta b > 0) \Rightarrow dir_k = \ 'l' ;$$

$$(\Delta a + \Delta b = 0) \Rightarrow dir_k = \ 'c' ;$$

$$(\Delta a + \Delta b < 0) \Rightarrow dir_k = 'r'.$$

# Power of a Trend Group

The ***power of a trend group*** is:

***slow ('<'), medium ('='), fast ('>')***

and it is defined by the signs of $\Delta a$ and $\Delta b$:

$$((\Delta a < 0)\ and\ (\Delta b > 0)) \Rightarrow pow_k = '<';$$

$$((\Delta a = 0)\ or\ (\Delta b = 0)) \Rightarrow pow_k = '=';$$

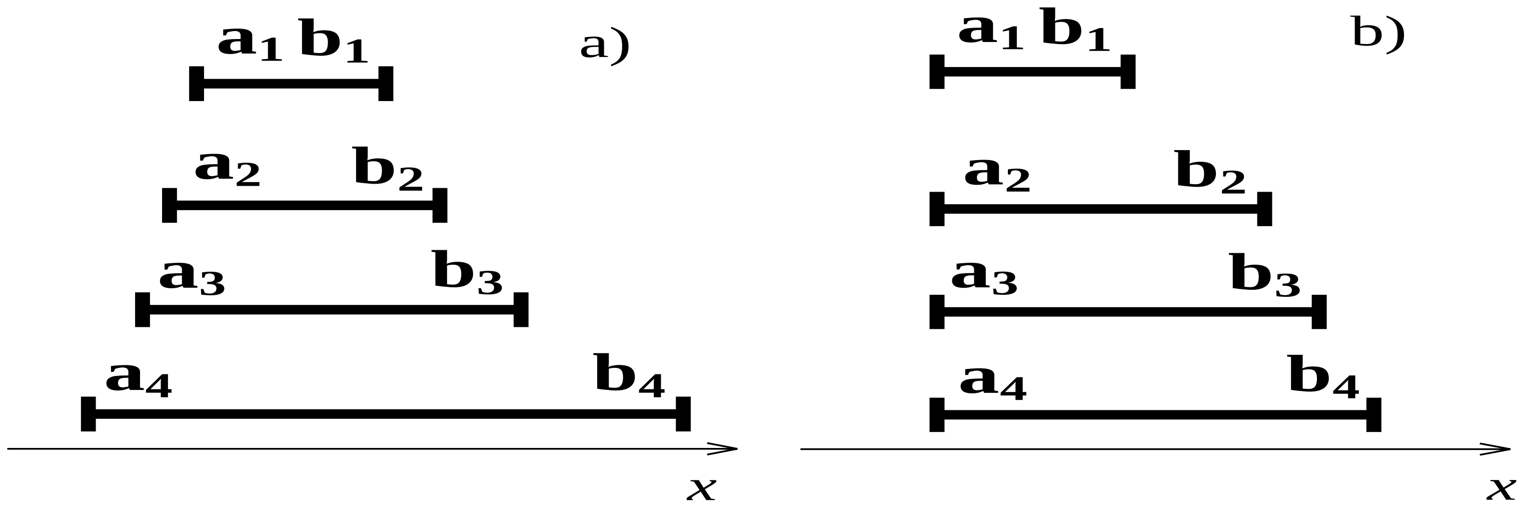
$$((\Delta a > 0)\ or\ (\Delta b < 0)) \Rightarrow pow_k\ =\ '>' .$$

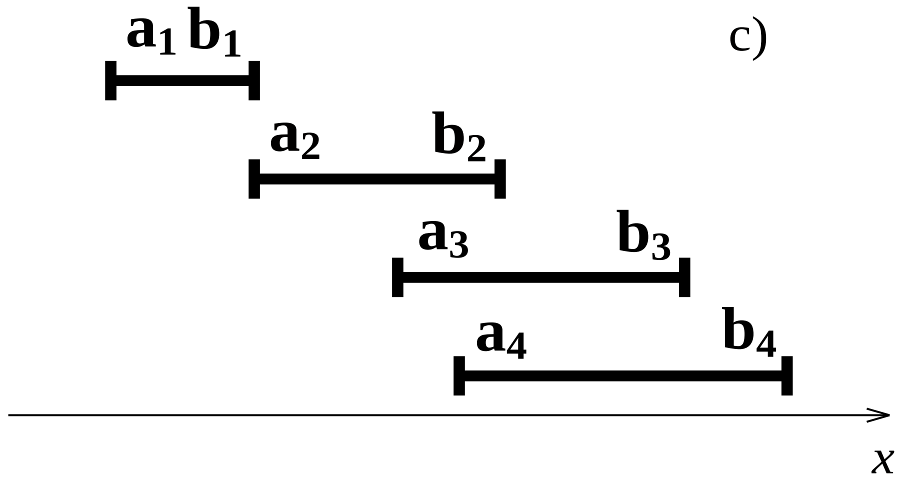
# Seven Groups of Uncertainty Trends

| ***Trend*** | **Direction→** | ***left*** | ***central*** | ***right*** |
|---|---|---|---|---|
| **Power↓** | Restrictions | $\Delta a + \Delta b > 0$ | $\Delta a + \Delta b = 0$ | $\Delta a + \Delta b < 0$ |
| ***slow*** | $(\Delta a < 0) and$ <br> $and (\Delta b > 0)$ | $L^{l<}$ | $L^{c<}$ | $L^{r<}$ |
| ***medium*** | $(\Delta a = 0) or$ <br> $or (\Delta b = 0)$ | $L^{l=}$ | does not exist | $L^{r=}$ |

| | | | | |
|---|---|---|---|---|
| ***fast*** | $(\Delta a > 0) or$ $or (\Delta b < 0)$ | $L^{l>}$ | does not exist | $L^{r>}$ |

# An Example of Left Trends: $L^{l<}, L^{l=}, L^{l>}$






# A Trend Keeping Theorem

***Let*** $$L_{[a_i, b_i]}^{L_{[a_j, b_j]}} = L_{[a_{res},\ b_{res}]},$$

***then*** the interval $L_{[a_{res}, b_{res}]}$ belongs to the same trend group as intervals $L_{[a_i, b_i]}, L_{[a_j, b_j]}$.

# A Support of a Trend

Let us suppose that ***n*** interval opinions $L_{[a_i, b_i]}$, $i = 1, \ldots n$ are divided into ***m*** trends $L_k$, $k = 1, \ldots m$.

The ***support*** $S_k$ for the trend $L_k$ is defined as follows:

$$S_k = q_{res}^k \cdot \sum_{\forall i, L_{[a_i, b_i]} \in L_k} \frac{1}{N_i}, \qquad (*)$$

where $q_{res}^k$ is the ***quality*** of the result $L_{[a_{res}^k, b_{res}^k]}$, $\boldsymbol{N_i}$ is the number of different trends that includes the opinion $L_{[a_i, b_i]}$.

*(*) As one can see the Definition gives more support for the trend that includes more intervals and the support of each interval is divided equally between all the trends that include this interval.*

# Deriving Resulting Interval from Several Trends

**Let** the set of original interval opinions $L = L_{[a_i, b_i]}$, $i = 1, \ldots, n$ consists of ***m*** different trends $L_k$, $k = 1, \ldots, m$ with their resulting interval opinions $L_{[a_{res}^k, b_{res}^k]}$ and support $S_k$.

**Then** the ***resulting opinion*** $^{L}\left[a_{res}^{L}, b_{res}^{L}\right]$ for the whole original set of interval opinions is derived as follows:

$$a_{res}^{L} = \frac{a_{res}^{1} \cdot S_1 + a_{res}^{2} \cdot S_2 + \ldots + a_{res}^{m} \cdot S_m}{S_1 + S_2 + \ldots + S_m};$$

$$b_{res}^{L} = \frac{b_{res}^{1} \cdot S_1 + b_{res}^{2} \cdot S_2 + \ldots + b_{res}^{m} \cdot S_m}{S_1 + S_2 + \ldots + S_m}. \qquad (**)$$

*(**) The resulting interval is expected to be closer to the result of those trends that have more support among the original set of intervals.*